

On the Thermodynamics of Hot Hadronic Matter

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Abstract

The equation of state of hot hadronic matter is obtained, by taking into account the contribution of the massive states with the help of the resonance spectrum $\tau(m) \sim m^3$ justified by the authors in previous papers. This equation of state is in agreement with that provided by the low-temperature expansion

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for the pion interacting gas. It is shown that in this picture the deconfinement phase transition is absent, in agreement with lattice gauge calculations which show the only phase transition of chiral symmetry restoration. The latter is modelled with the help of the restriction of the number of the effective degrees of freedom in the hadron phase to that of the microscopic degrees of freedom in the quark-gluon phase, through the corresponding truncation of the hadronic resonance spectrum, and the decrease of the effective hadron masses with temperature, predicted by Brown and Rho. The results are in agreement with lattice gauge data and show a smooth crossover in the thermodynamic variables in a temperature range ~ 50 MeV.

Key words: hot hadronic matter, resonance spectrum, equation of state, quark-gluon plasma, chiral symmetry restoration

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1 Introduction

One of the main goals of experiments with high-energy nuclear collisions is to produce and to study hadronic matter, in particular, trying to reach conditions at which the phase transitions into the quark-gluon plasma phase can take place [1–5]. The physics of hot hadronic matter has not been studied much, although such matter is already produced in current experiments in high-energy physics.

The experimental data on multiple hadron production obtained in recent years are in agreement with the main consequences of the theory formulated by Landau [6] over 40 years ago. However, with a sufficiently quantitative approach, it becomes necessary to consider a number of physical effects which bring about certain modifications of the results obtained in the fundamental work [6]. For instance, the solution of the equations of motion obtained by Landau differs from a numerical calculation of Milekhin [7], as a result of an inaccurate estimate. A more accurate analytic solution was given in [8].

The equation of state in Landau's work is taken to be $p = \rho/3$ (where p is the pressure and ρ is the energy density), corresponding to an ultra-relativistic gas. However, the CERN and FNAL colliders provide proton-antiproton collisions with the total center-of-mass energy of the order of 1 TeV, corresponding to initial temperatures $T \lesssim 10$ GeV for which the interaction of the hadrons is strong and has mainly a resonant character, the masses of the resonances being comparable with the temperature. Thus, hadronic matter under these conditions is neither an ideal nor an ultrarelativistic gas.

Corrections in the equation of state due to the interaction of the hadrons have been discussed in the literature for the last three decades [1,5,8-16]. These considerations

were based mostly on a phenomenological model in which the Landau theory is applied to all particles except the leading ones, i.e., the fragments of the initial particles, whose characteristics are taken directly from experimental data. The framework for the latter considerations is the QCD phase transition of hadronic matter into the quark-gluon plasma.

Hot hadronic matter, in which volumes per particle are a few cubic fermis, is certainly made out of individual hadrons. It is clear that, at low temperatures $T \ll m_\pi$, one has a very rare (and therefore ideal) gas of the lightest hadrons, the pions. As the temperature is raised and the gas becomes more dense, one should take into account interactions among the particles. This was done using the following three approaches: (i) the low-temperature expansion, (ii) the resonance gas, (iii) the quasi-particle gas.

The first approach for the pion gas is based on the Weinberg theory of pion interactions [17], which uses the non-linear Lagrangian containing all processes quadratic in pion momentum. Its first application to calculation of the thermodynamic parameters of the pion gas was done in ref. [1]. One of the important consequences of this work was that, after isospin averaging, all corrections quadratic in momenta cancel each other, and corrections proportional to the pion mass (Weinberg $\pi\pi$ scattering lengths) are nearly compensated, producing negligible corrections at the 1% level. Corrections of the second order in the Weinberg Lagrangian were also estimated in [1], and a much more systematic study of the problem including quartic terms in the mesonic Lagrangian were made in ref. [18]. It has been shown that taking into account the pion rescattering leads to corrections quartic in momenta which, after isospin averaging, give a nonzero correction to the pressure, providing a scale for the temperatures at which deviations from the ideal pion gas formula become noticeable:

$$p = \frac{\pi^2}{30} T^4 \left[1 + \left(\frac{T}{T_{int}} \right)^4 \right], \quad T_{int} \simeq 150 \text{ MeV}. \quad (1)$$

Without going into discussion of these works we remark that the applicability of this approach is limited by temperatures $T < 100$ MeV, for which typical collision energies are significantly below resonance. However, such T are lower than even the lowest temperature available in experiments, because the so-called break-up temperatures are typically $T \simeq 120 - 150$ MeV.

The idea of resonance gas was first suggested by Belenky and Landau [19], as early as 1956. They used the Beth-Uhlenbeck method [20], well known in the theory of the non-ideal gases [21], the main idea of which consists in the calculation of the number of states within the normalization volume with the scattering phase shifts taken into account. At large distances r between the particles, their wave function is given by $\psi_\ell(r) \approx \sin(pr + \delta_\ell)/r$, and the boundary condition $\psi_\ell(R) = 0$ at the boundary of the normalization volume picks out the states with momenta $pR + \delta_\ell(p) = \pi n$. The replacement of the summation over n in the statistical sum by an integral over p gives

rise to the following expression (Z is only a part of the statistical sum connected with the relative motion of the particles):

$$Z = \frac{1}{\pi} \sum_{\ell} (2\ell + 1) \int_0^{\infty} dp \left(R + \frac{d\delta_{\ell}(p)}{dp} \right) e^{-E(p)/T}. \quad (2)$$

If there is a resonance in the scattering at a certain momentum p_0 , then $\delta_{\ell}(p)$ changes rapidly in the neighborhood of p_0 . It is easily seen that the contribution of the resonance to the integral (2) is the term $\exp(-E(p_0)/T)$, the same as for a bound state with that energy. Therefore, resonances and stable particles should be taken into account on an equal footing in the thermodynamic characteristics of hot hadronic matter; e.g., the formulas for the pressure and energy density in a resonance gas read (we neglect the chemical potential for simplicity)

$$p = \sum_i p_i = \sum_i g_i \frac{m_i^2 T^2}{2\pi^2} \sum_{r=1}^{\infty} (\pm 1)^{r+1} \frac{K_2(r m_i/T)}{r^2}, \quad (3)$$

$$\rho = \sum_i \rho_i, \quad \rho_i = T \frac{dp_i}{dT} - p_i, \quad (4)$$

where $+1$ (-1) corresponds to the Bose-Einstein (Fermi-Dirac) statistics, and g_i are the corresponding degeneracies (J and I are spin and isospin, respectively),

$$g_i = \begin{cases} (2J_i + 1)(2I_i + 1) & \text{for non-strange mesons} \\ 4(2J_i + 1) & \text{for strange (K) mesons} \\ 2(2J_i + 1)(2I_i + 1) & \text{for baryons} \end{cases}$$

These expressions may be rewritten with the help of a *resonance spectrum*,

$$p = \int_{m_1}^{m_2} dm \tau(m) p(m), \quad p(m) \equiv \frac{m^2 T^2}{2\pi^2} \sum_r (\pm 1)^{r+1} \frac{K_2(r m/T)}{r^2}, \quad (5)$$

$$\rho = \int_{m_1}^{m_2} dm \tau(m) \rho(m), \quad \rho(m) \equiv T \frac{dp(m)}{dT} - p(m), \quad (6)$$

normalized as

$$\int_{m_1}^{m_2} dm \tau(m) = \sum_i g_i, \quad (7)$$

where m_1 and m_2 are the masses of the lightest and heaviest species, respectively, entering the formulas (3),(4).

In both the statistical bootstrap model [9, 10] and the dual resonance model [11], a resonance spectrum takes on the form

$$\tau(m) \sim m^a e^{m/T_0}, \quad (8)$$

where a and T_0 are constants. The treatment of hadronic resonance gas by means of the spectrum (8) leads to a singularity in the thermodynamic functions at $T = T_0$ [9, 10] and, in particular, to an infinite number of the effective degrees of freedom in the hadron phase, thus hindering a transition to the quark-gluon phase. Moreover, as shown by Fowler and Weiner [12], an exponential mass spectrum of the form (8) is incompatible with the existence of the quark-gluon phase: in order that a phase transition from the hadron phase to the quark-gluon phase be possible, the hadronic spectrum cannot grow with m faster than a power.

In 1972, Shuryak [8], in place of (8), used the simple power parametrization

$$\tau(m) \sim m^k, \quad (9)$$

which “describes experiment in the mass region $0.2 - 1.5$ GeV for $k \approx 3$.” Since $\tau(m)$ is a rapidly growing function, the main contribution to integrals of type (10) is given by the mass region in which $\exp(m/T) \gg 1$, and the difference in properties of bosons and fermions is irrelevant. Upon substitution of (9) into (5) and (6) one finds that the pressure and energy density are proportional to T^{k+5} . Therefore, the velocity of sound c_s , defined by

$$c_s^2 = \frac{dp}{d\rho}, \quad (10)$$

turns out to be a temperature independent constant, and the equation of state belongs to the class considered by Milekhin [22]. For this case, the expressions for energy and pressure are as follows,

$$\rho = \lambda T^{k+5}, \quad p = \frac{\lambda}{k+4} T^{k+5}, \quad c_s^2 = \frac{1}{k+4}, \quad (11)$$

where λ is a certain constant. Consequently, for $k \approx 3$ [8], $c_s^2 \approx 0.14$.

In 1975, Zhurov and Shuryak [13] have calculated the velocity of sound, Eq. (10), directly from Eqs. (5),(6), and found that at first it increases with T very quickly and then saturates at the value of $c_s^2 \simeq 1/3$ if only the pions are taken into account, and $c_s^2 \simeq 1/5$ if resonances up to $M \sim 1.7$ GeV are included, hence favoring $k \approx 1$ in (11) and the equation of state $p, \rho \sim T^6$, $p = \rho/5$. Such an equation of state was suggested by Shuryak in his book [5] of 1988 (and referred to also in [1]), by fitting the real resonance spectrum,

$$p \simeq (20 \text{ GeV}^{-2}) T^6, \quad (12)$$

and called the “realistic” equation of state of hot hadronic matter. It gives $c_s^2 = 0.20$, in agreement with the theoretical models considered in earlier works [23, 24, 25].

In ref. [14] Shuryak, in order to describe the behavior of the energy density with temperature $\rho \propto T^6$, proposed to consider hot hadronic matter as a gas of quasi-particles, which have quantum numbers of the original mesons, but with dispersion

relations modified by the interaction with matter (similarly to Landau's idea of "rotons", to explain the growth of the energy density of liquid ${}^4\text{He}$ with temperature more rapidly than T^4). Since in such an approach hadrons are included as physical degrees of freedom of hot hadronic matter, they are assumed not to be absorbed too strongly, i.e., to be good quasiparticles. In ref. [14] an attempt was made to guess what these dispersion relations should be, in order to obtain the expected behavior of the energy density with temperature. In [16] these dispersion relations were calculated in explicit form.

It may seem strange that the different approaches (i.e., (i) and (ii) out of the three mentioned above, the approach (iii) was invented in order to justify the results given by (ii)) should lead to different results, viz., Eqs. (1) and (12); moreover, it is not clear which result, if it does, corresponds to the genuine thermodynamics of the hadronic resonance gas. In this article we shall show that with the correct form of the hadronic resonance spectrum which has been established by the authors in a series of papers [26, 27, 28], the equation of state of hot hadronic matter takes on the form (1), but reduces to (12) in the case of only lower mass resonances being taken into account.

2 Hadronic resonance spectrum and the equation of state

In our previous work [26] we considered a model for a transition from a phase of strongly interacting hadron constituents, described by a manifestly covariant relativistic statistical mechanics which turned out to be a reliable framework in the description of realistic physical systems [29], to the hadron phase described by a resonance spectrum, Eqs. (5),(6). An example of such a transition may be a relativistic high temperature Bose-Einstein condensation studied by the authors in ref. [30], which corresponds, in the way suggested by Haber and Weldon [31], to spontaneous flavor symmetry breakdown, $SU(3)_F \rightarrow SU(2)_I \times U(1)_Y$, upon which hadronic multiplets are formed, with the masses obeying the Gell-Mann–Okubo formulas [32]

$$m^\ell = a + bY + c \left[\frac{Y^2}{4} - I(I+1) \right]; \quad (13)$$

here I and Y are the isospin and hypercharge, respectively, ℓ is 2 for mesons and 1 for baryons, and a, b, c are independent of I and Y but, in general, depend on (p, q) , where (p, q) is any irreducible representation of $SU(3)$. Then the only assumption on the overall degeneracy being conserved during the transition leads to the unique form of a resonance spectrum in the hadron phase:

$$\tau(m) = Cm, \quad C = \text{const}, \quad (14)$$

in agreement with the results of Zhirov and Shuryak [13] found on a phenomenological ground. We have checked the coincidence of the results given by a linear spectrum (14) with those obtained directly from Eq. (3) for the actual hadronic species with the corresponding degeneracies, for all well-established hadronic multiplets, both mesonic and baryonic, and found it excellent [26]. Therefore, the fact established theoretically that a linear spectrum is the actual spectrum in the description of individual hadronic multiplets, finds its experimental confirmation as well. In our recent papers [27, 28] we have shown that a linear spectrum of an individual meson nonet is consistent with the Gell-Mann–Okubo mass formula

$$m_1^2 + 3m_8^2 = 4m_{1/2}^2 \quad (15)$$

(in fact, this formula may be derived with the help of a linear spectrum [27]), and leads to an extra relation for the masses of the isoscalar states, $m_{0'}$ and $m_{0''}$, (of which $0'$ belongs to a mostly octet),

$$m_{0'}^2 + m_{0''}^2 = m_0^2 + m_8^2 = 2m_{1/2}^2, \quad (16)$$

with $m_1, m_{1/2}, m_8, m_0$ being the masses of the isovector, isospinor, and isoscalar octet and singlet states, respectively, which for an almost ideally mixed nonet reduces to [27, 28]

$$m_{0''}^2 \simeq m_1^2, \quad m_{0'}^2 \simeq 2m_{1/2}^2 - m_1^2. \quad (17)$$

We have checked the relation (17) in ref. [28] and shown to hold with an accuracy of up to $\sim 3\%$ for all well-established nonets. In ref. [27] we have generalized a linear spectrum to the case of four quark flavors and derived the corresponding Gell-Mann–Okubo mass formula for an $SU(4)$ meson hexadecuplet, in a good agreement with the experimentally established masses of the charmed mesons. In ref. [33] we have applied a linear spectrum to the problem of establishing the correct $q\bar{q}$ assignment for the problematic meson nonets, like the scalar, axial-vector and tensor ones, and separating out non- $q\bar{q}$ mesons. In this paper we shall apply a resonance spectrum to the derivation of the thermodynamic characteristics of hot hadronic matter.

Let us first consider the case of the meson resonances alone. For this case, the normalization constant C of a linear spectrum (14) was established in ref. [26]: for a nonet, one has 9 isospin degrees of freedom lying in the interval $(m_{0''} \simeq m_1, m_{0'})$. Therefore, Eq. (7) gives

$$C \int_{m_{0''}}^{m_{0'}} dm \, m = 9,$$

and hence

$$C = \frac{18}{m_{0'}^2 - m_{0''}^2} \equiv \frac{18}{\Delta} \simeq 27 \text{ GeV}^{-2}, \quad (18)$$

where the difference $\Delta \equiv m_{0'}^2 - m_{0''}^2$ is determined by a distance between the parallel Regge trajectories for the ω and ϕ resonances, which are described by the straight

lines $J = 0.59 + 0.84M^2$ and $J = 0.04 + 0.84M^2$, respectively, so that $\Delta \simeq 0.65 \text{ GeV}^2$. We note further that the meson nonets may be arranged in the pairs of nonets which have equal parity but different spins (which differ by 2), e.g.,¹

$$1 \ ^3P_0 \ J^{PC} = 0^{++}, \ a_0(1320), \ f_0(1300), \ f_0(1525), \ K_0^*(1430),$$

$$1 \ ^3P_2 \ J^{PC} = 2^{++}, \ a_2(1320), \ f_2(1270), \ f_2'(1525), \ K_2^*(1430),$$

$$1 \ ^3D_1 \ J^{PC} = 1^{--}, \ \rho(1700), \ \omega(1600), \ K^*(1680), \ (\text{no } \phi \text{ candidate}),$$

$$1 \ ^3D_3 \ J^{PC} = 3^{--}, \ \rho_3(1700), \ \omega_3(1600), \ \phi_3(1850), \ K_3^*(1780),$$

$$3 \ ^1S_0 \ J^{PC} = 0^{-+}, \ \pi(1770), \ \eta(1760), \ K(1830), \ (\text{no } \eta' \text{ candidate}),$$

$$1 \ ^1D_2 \ J^{PC} = 2^{-+}, \ \pi_2(1670), \ K_2(1770), \ (\text{no } \eta, \eta' \text{ candidates}),$$

and occupy the mass interval of an individual nonet but have 18 isospin degrees of freedom in this interval, i.e., twice as much as that for an individual nonet. Moreover, as the temperature gets closer to the critical one of chiral symmetry restoration, we expect the chiral partners (the states with equal isospin but different parity) have equal masses and form parity doublets. The work of DeTar and Kogut [34] shows convincingly that the “screening masses” of chiral partners are different below and become equal above a common T_c . This work was carried out for four quark flavors. Similar results were obtained for two flavors by Gottlieb *et al.* [35]. In these calculations, the chiral partners were (π, σ) , (ρ, a_1) and $(N(\frac{1}{2}+), N(\frac{1}{2}-))$. Thus, we expect the correct density of states per unit mass interval to be twice as much as that for an individual nonet, and hence, the correct normalization constant is

$$C \simeq 54 \text{ GeV}^{-2}. \quad (19)$$

Once the mass spectrum of a nonet (with a given fixed spin) is established to be linear, one may take into account different nonets with different spins in Eqs. (3),(4). As shown in ref. [26], since the particle spin is related to its mass, $J_i \sim \alpha' m_i^2$, α' being a universal Regge slope, the spin degeneracy turns out to be proportional to the mass squared, and the account for different nonets results in the following mass spectrum,

$$\tau'(m) = C' m^3, \quad C' = 2\alpha' C \simeq 90 \text{ GeV}^{-4}, \quad (20)$$

which is the actual resonance spectrum of hadronic matter and should lead, through (11), to the equation of state

$$p, \rho \sim T^8, \quad p = \rho/7. \quad (21)$$

Bebie *et al.* [36] have calculated the ratio ρ/p directly from Eqs. (3),(4), with all known hadron resonances with the masses up to 2 GeV taken into account, and found that the curve ρ/p first decreases very quickly and then saturates at the value of $\rho/p \simeq 7$, as read off from Fig. 1 of ref. [36], in agreement with (21).

¹For the scalar meson nonet, we use the $q\bar{q}$ assignment suggested by the authors in ref. [28, 33].

In order to show that the obtained normalization constant is correct, we note that the number of states with the masses up to M , given by the mass spectrum (20), is

$$N(M) = \frac{C' M^4}{4} \simeq 22.5 (M, \text{ GeV})^4. \quad (22)$$

For, e.g., $M = 1.25$ Eq. (22) gives

$$N(1.25) \simeq 55. \quad (23)$$

The masses up to 1.25 GeV have the members of the pseudoscalar and vector meson nonets, and the $h_1(1170)$, $b_1(1235)$ and $a_1(1260)$ mesons, the mass of the latter was indicated by the recent Particle Data Group as 1.23 GeV [37]. We do not include the scalar mesons $a_0(980)$ and $f_0(980)$ which seem to be non- $q\bar{q}$ objects [33], but may include the $f_0(1300)$ meson which has the mass lying in the interval 1 – 1.5 GeV, according to the recent Particle Data Group. Thus, we have 9+27+1+9+9+1=56 actual mesonic species having the masses up to 1.25 GeV, in excellent agreement with (23).

For $M = 1.7$ GeV, Eq. (22) gives

$$N(1.7) \simeq 188. \quad (24)$$

As seen in the Meson Summary Table [38], the masses up to 1.7 GeV have the members of the following nonets: 1^1S_0 , 1^3S_1 , 1^1P_1 , 1^3P_0 , 1^3P_1 , 1^3P_2 , 2^1S_0 , 2^3S_1 . Therefore, one has

$$(20 \text{ spin states}) \times (9 \text{ isospin states}) = 180 \text{ states}, \quad (25)$$

in good agreement with the result (24) given by a cubic spectrum.

For $M = 2$ GeV, Eq. (22) gives

$$N(2) \simeq 360. \quad (26)$$

The masses up to 2 GeV have the members of all the nonets indicated in [38] except for the 1^3F_4 and 2^3P_2 nonets. In this case, one has

$$(41 \text{ spin states}) \times (9 \text{ isospin states}) = 369 \text{ states}, \quad (27)$$

again in good agreement with the result (26) given by a cubic spectrum. Thus, we consider the cubic spectrum (20) as granted by the actual experimental meson spectrum.

Now, as the actual meson resonance spectrum is established, one uses this spectrum in Eqs. (5),(6), in order to obtain the equation of state of hot hadronic (mesonic) matter. First we note that one may approximate the particle statistics by the Maxwell-Boltzmann one, because of the richness of the spectrum $\tau(m) \sim m^3$

at large m ; then the sum over r in Eqs. (5),(6) may be approximated by $\pi^4/90 = \zeta(4) \equiv \sum_r 1/r^4$, which is the asymptotic form of this sum for $T \gg m$. One obtains, therefore,

$$p = \frac{\pi^2}{30} T^4 + \frac{C' \pi^2}{180} T^4 \int dm m^3 \left(\frac{m}{T}\right)^2 K_2\left(\frac{m}{T}\right), \quad (28)$$

$$\rho = T \frac{dp}{dT} - p,$$

where we have separated out the contribution of the pions which may well be treated as massless at temperatures ~ 150 MeV, and taken into account the remaining particle species by an integration over m with the resonance spectrum (20), which therefore has the lower limit ~ 0.5 MeV $\simeq m_K$. We note that this treatment of the hadronic resonance gas corresponds, in view of (3),(5), to a collection of free (non-interacting) particles, which is completely justified at lower temperatures, since chiral symmetry suppresses the interactions of low energy Goldstone bosons both among themselves and with massive hadrons, but becomes an approximation as the temperature increases. Since the main contribution to integrals of the type (28) is given by the mass region in which $m \gg T$, one may extend the upper limit of integration to infinity and neglect the lower limit, and obtain, through the formula [39]

$$\int_0^\infty dx x^\mu K_\nu(ax) = 2^{\mu-1} a^{-\mu-1} \Gamma\left(\frac{1+\mu+\nu}{2}\right) \Gamma\left(\frac{1+\mu-\nu}{2}\right),$$

$$p \simeq \frac{\pi^2}{30} T^4 \left[1 + \left(\frac{T}{160 \text{ MeV}}\right)^4 \right], \quad (29)$$

$$\rho \simeq \frac{\pi^2}{10} T^4 \left[1 + \frac{7}{3} \left(\frac{T}{160 \text{ MeV}}\right)^4 \right]. \quad (30)$$

The formulas (29),(30) represent the equation of state of hot mesonic matter.

If one restricts himself to the lower mass resonances (i.e., the lower spin, $J = 0, 1$, nonets) alone, one may use the linear spectrum (14) with C defined in (19) and obtain

$$p = \frac{C \pi^2}{180} T^4 \int dm m \left(\frac{m}{T}\right)^2 K_2\left(\frac{m}{T}\right) \simeq (23 \text{ GeV}^{-2}) T^6, \quad (31)$$

in apparent agreement with the Shuryak's "realistic" equation of state (12).

The formulas (29),(30) have been obtained for the meson resonances alone. There is no difficulty of principle to consider the baryon resonances in a similar way, with the inclusion of two chemical potentials, for both conserved net baryon number and strangeness. As we have checked in ref. [26], a mass spectrum of the $SU(3)$ baryon multiplets is linear, as well as for the meson nonets, although to establish its correspondence to the Gell-Mann–Okubo formulas is more difficult than for a meson nonet, since these formulas are linear in mass for baryons (more detailed discussion is given in ref. [26]). Recent result of Kutasov and Seiberg [40] shows that the numbers of

bosonic and fermionic states in a non-supersymmetric tachyon-free string theory must approach each other as increasingly massive states are included. The experimental hadronic mass spectrum shows that in the mass range $\sim 1.2 - 1.7$ GeV, the number of baryon states nearly keeps pace with that of meson states [41] (and, therefore, is well described by the same cubic spectrum as for the mesons, Eq. (20)). Above ~ 1.7 GeV, the number of the observed baryons begins to outstrip that of the mesons, and then greatly surpasses the latter at higher energies (indicating, therefore, that the baryon resonance spectrum grows faster than (20) in this mass region, since the cubic spectrum (20) describes the meson resonances well, up to, at least, 2 GeV, as we have seen in Eqs. (23)-(27)). The explanation of this behavior of the experimental resonance spectrum was found by Cudell and Dienes in a naive hadron-scale string picture [42]: the ratio of the numbers of the baryon and meson states should, in fact, oscillate around unity, with the mesons favored first, then baryons, then mesons again, etc. Keeping in mind this picture, we may assume that the “in-average” baryon resonance spectrum has the same form, Eq. (20), as the meson resonance one. If one now neglects, for simplicity, the baryon number and strangeness chemical potentials (i.e., considers the case of both zero net baryon number and strangeness), and takes into account the baryon resonances along with the meson ones by the mass spectrum (20), one will obtain the same formulas (28) but with an extra term in the r.h.s. which differs from the second term of Eq. (28) by the factor $7/8$ (since for the baryons, the sum over r in Eqs. (5),(6) is approximated by $7\pi^2/720 = 7/8 \zeta(4) = \sum_r (-1)^{r+1}/r^4$). These formulas will further reduce to the relations

$$p \simeq \frac{\pi^2}{30} T^4 \left[1 + \left(\frac{T}{140 \text{ MeV}} \right)^4 \right], \quad (32)$$

$$\rho \simeq \frac{\pi^2}{10} T^4 \left[1 + \frac{7}{3} \left(\frac{T}{140 \text{ MeV}} \right)^4 \right], \quad (33)$$

which represent the equation of state of hot hadronic (both mesonic and baryonic) matter.

Comparison of Eqs. (29),(32) with (1) shows that the agreement between the both approaches, (i) the low-temperature expansion for the pion interacting gas, and (ii) the hadronic resonance gas, is very good, if the latter is treated with the help of the realistic mass spectrum, Eq. (20).

We shall use Eq. (1) and the corresponding formula for the energy density,

$$\rho \simeq \frac{\pi^2}{10} T^4 \left[1 + \frac{7}{3} \left(\frac{T}{150 \text{ MeV}} \right)^4 \right], \quad (34)$$

as the equation of state of hot hadronic matter, in agreement with the results (29),(32) provided by the hadronic resonance spectrum, Eq. (20). It is seen in (34) that the ratio ρ/T^4 rapidly grows with T , proportionally to T^4 . This behavior is to be expected,

since the value $\rho/T^4 = \pi^2/10 \simeq 1$ characteristic of a gas of the pions is small compared to the value $\rho/T^4 = 47.5\pi^2/30 \approx 15.6$ which pertains to the plasma of free quarks and gluons (47.5 being the number of degrees of freedom in the plasma with $N_c = N_f = 3$). The energy density stored in the excited states reaches the energy density of the pionic component at $T \simeq 120$ MeV, as follows from (34), in agreement with the value $\simeq 125$ MeV obtained by Bebie *et al.* [36] through direct calculation with the help of Eq. (4). The quark-gluon plasma value $\rho \simeq 15.6 T^4$ is reached at $T \simeq 235$ MeV. However, the energy densities of the hadron and quark-gluon phases need not match at the critical point, since the phase transition may liberate latent heat. The only pressure in both phases should match at the critical temperature, for which, therefore, a naive estimate may be obtained by equating (1) with the plasma's $p = 47.5\pi^2/90 T^4$: $T_c \simeq 290$ MeV. The phase transition from the hadron to the quark-gluon phase is expected to occur at much lower temperature. Indeed, the approximation of the hadronic resonance gas by a collection of free particles is justified up to temperatures ~ 150 MeV. Above this point, the interaction among the constituents of the gas rapidly grows with T and inelastic collisions become increasingly important. Moreover, direct calculation, with the help of Eqs. (3),(4), made by Bebie *et al.* [36] shows that the mean distance between two particles of the gas reaches the value $d = 1$ fm at $T \simeq 190$ MeV, indicating that Eqs. (3),(4) pack the hadrons very densely there and suggesting, therefore, that color cannot remain confined much beyond this point. Thus, the equation of state (1),(34) may be an adequate representation of (3),(4) up to temperatures ~ 190 MeV. The phase transition to the quark-gluon plasma is expected to occur at the same temperatures, $\sim 180 - 190$ MeV.² We conclude, therefore, that the equation of state (1),(34) is the realistic equation of state of hot hadronic matter in the whole temperature range of the latter: $0 - T_c$, with T_c being the critical temperature of the hadron to quark-gluon phase transition. In the following section we shall build a model for the thermodynamics of this transition.

3 Phase transition to the quark-gluon plasma

In the simplest schematic model the quark-gluon plasma is described by the equation of state

$$p = \frac{47.5\pi^2}{90}T^4 - B, \quad \rho = \frac{47.5\pi^2}{30}T^4 + B, \quad (35)$$

where B is the bag constant, and 47.5 the effective degeneracy of 8 gluons and 3 massless quarks. In going from the hadron phase (and its nonperturbative vacuum)

²The value of the critical temperature of the chiral symmetry restoration transition, to be associated with the hadron to quark-gluon one, calculated with the contribution of the massive states being taken into account, is [43] ~ 190 MeV if only the meson resonances, and ~ 175 MeV if both the baryon and meson resonances are considered, in agreement with the results obtained previously by Gerber and Leutwyler [18].

to the deconfined quark-gluon phase (with perturbative vacuum) we restore conformal invariance. Consequently [44], we should use the value of B given by the conformal anomaly:

$$B = \frac{1}{4} \langle 0 | \theta_\mu^\mu | 0 \rangle = -\frac{1}{4} \langle 0 | \frac{\beta(g)}{2g} (G_{\mu\nu}^a)^2 | 0 \rangle. \quad (36)$$

Here θ_μ^μ is the trace of the energy-momentum tensor and $\langle 0 | (G_{\mu\nu}^a)^2 | 0 \rangle$ is the gluon condensate. The β -function of the renormalization group is given by [45]

$$\beta(g) = -\frac{g^3}{16\pi^2} \left(11 - \frac{2}{3} N_f \right) \quad (37)$$

for N_f flavors. The value of the gluon condensate,

$$\frac{g^2}{4\pi^2} \langle 0 | (G_{\mu\nu}^a)^2 | 0 \rangle \simeq (330 \text{ MeV})^4, \quad (38)$$

is well determined by the states of charmonium using QCD sum rules [46]. For three flavors, we then find

$$B^{1/4} \simeq 240 \text{ MeV}, \quad (39)$$

and Eqs. (35) reduce to

$$p \simeq \frac{47.5\pi^2}{90} T^4 \left[1 - \left(\frac{160 \text{ MeV}}{T} \right)^4 \right], \quad \rho \simeq \frac{47.5\pi^2}{30} T^4 \left[1 + \frac{1}{3} \left(\frac{160 \text{ MeV}}{T} \right)^4 \right]. \quad (40)$$

If the hadron phase is described, e.g., by Eqs. (29),(30), a simple equating of the pressure in both phases (Eqs. (29) and (40)) gives two real positive roots:

$$T_1 \approx 166 \text{ MeV}, \quad T_2 \approx 308 \text{ MeV}. \quad (41)$$

In the temperature range $T_1 \leq T \leq T_2$, the curve $p = p(T)$ for the quark-gluon phase goes above the corresponding curve for the hadron phase, but at $T > T_2$, the situation changes: the quark-gluon curve goes below the hadron one. In this picture, therefore, the hadron phase is again thermodynamically favored at high temperatures, and the existence of the quark-gluon phase is restricted to a limited range of temperatures, which is clearly unphysical. The reason for such unphysical behavior predicted by Eqs. (29),(40) is, except for the applicability of Eqs. (29),(30) limited by $T \sim 190$ MeV, the effective degeneracy in the hadron phase which grows, according to (29), as $g_{eff} \sim T^4$. As shown by Brown *et al.* [44], the free energy of an interacting system of massless microscopic constituents (and hence, of the effective degrees of freedom which are intended to mimic these interactions) should be greater than the free energy of the noninteracting microscopic constituents at any temperature. When these interactions are realized in terms of massless hadrons, this constraint implies that the number of the effective (hadronic) degrees of freedom should always be less than 47.5

and that it should *approach* 47.5 in the high-temperature limit. In other words, the number of degrees of freedom in the effective excitations (hadrons) should not exceed that of the underlying quarks and gluons. Specifically, the hadronic resonances are intended to simulate attractive channels of the microscopic constituents. The stability of the trivial microscopic vacuum at zero temperature against the production of infinitely many particles requires the existence of repulsive channels for the microscopic constituents, which is not taken into account in Eqs. (1),(29),(32). At the one-body level, such repulsion is most easily simulated by associating an excluded volume with each hadron [44]. However, the most naive assumption of a temperature independent excluded volume represents an “overkill” in the high-temperature description of the now massless hadrons, since it leads to an effective degeneracy which vanishes as the temperature grows up [44]. A more realistic description of the hadron phase can be obtained using an excluded volume which decreases like $1/T^3$ for large T , which enforces the limitation of the number of the effective degrees of freedom by a fixed number, which may be made taken a desired value (e.g., 47.5) by the corresponding choice of the parametrization for a temperature dependent excluded volume [44]. An alternative approach is to restrict ourselves to a finite number of hadronic states, which should be chosen to be 47.5, since, according to the arguments given above, we should have 47.5 effective massless degrees of freedom in the hadron phase at high temperature. For our present semiquantitative purposes, this approach provides an adequate alternative to the use of excluded volumes.

The most natural way to restrict the effective degeneracy in the hadron phase (to 47.5) is truncating the resonance spectrum (20) when the number of the effective hadronic degrees of freedom is 47.5. This makes certain sense from the physical point of view, similar to the vibrations of a Debye solid where the phonon spectrum is truncated when the number of phonons is $3N$, with N being the number of atoms. It is seen in (22) that $M \simeq 1.2$ GeV corresponds to $N = 47.5$; we consider, therefore, the following expression for the pressure in the hadron phase with 47.5 effective degrees of freedom:

$$p = \frac{\pi^2}{30}T^4 + \frac{C'\pi^2}{180}T^4 \int_{0.5}^{1.2} dm m^3 \left(\frac{m}{T}\right)^2 K_2\left(\frac{m}{T}\right), \quad (42)$$

where we have, as previously, separated out the contribution of (massless) pions. It is now seen that at $T \gtrsim 1.2$ GeV, this formula reduces, through $K_2(x) \sim 2/x^2$, $x \ll 1$, to

$$p = \frac{\pi^2}{30}T^4 + \frac{C'\pi^2}{90}T^4 \int_{0.5}^{1.2} dm m^3; \quad (43)$$

since $C' \int_{0.5}^{1.2} dm m^3 = 44.5$ ($= 47.5 - 3$ of the pions), it follows from (43) that at high enough temperature,

$$p = \frac{47.5\pi^2}{90}T^4, \quad (44)$$

i.e., we have 47.5 effective massless degrees of freedom in the hadron phase. Further comparison of Eqs. (40) and (44) shows that the transition to the deconfined

quark-gluon phase is still prohibited by the bag constant B : the pressure curve (40) goes below the corresponding (44) at high T , and hence the hadron phase is thermodynamically favored there, even if the curves (40) and (44) intersect at some lower temperature. Thus, one sees that even in the model with the restricted effective degeneracy in the hadron phase, the deconfinement transition is *absent*. This conclusion is in agreement with lattice gauge calculations which show the only phase transition, viz., the restoration of chiral symmetry [34, 35], not deconfinement. As stressed out in ref. [44], the deconfinement transition for pure glue, used in lattice calculations as a model problem for many years, is a fictitious transition; it is completely changed by the introduction of light mass quarks. Moreover, the results of lattice gauge calculations indicate that the gluon condensate is essentially unchanged during the chiral restoration transition [47]. Even at a temperature as high as ~ 290 MeV, lattice calculations find about half as much gluon condensate as at zero temperature [48]. Apparently, the conformal anomaly (and the bag constant associated with the former) is relatively unaffected by what happens to the quark condensate. The same conclusion was reached by Brown [49] in the connection with the variations of a chemical potential rather than temperature. Thus, even at temperatures well above the critical one of chiral symmetry restoration, T_χ (~ 190 MeV), gluons remain condensed. Consequently, although the mesons can be regarded as quark-antiquark pairs, each quark must be connected with an antiquark by a “string” (i.e., a line integral of the gauge vector potential), in order to preserve gauge invariance. It is difficult to include the important consequences of such quark-antiquark correlations in the thermodynamics of the gluon condensed system, unless one keeps regarding the mesons as the correct effective degrees of freedom even above T_χ . Since our equation of state (42) does regard the mesons as the effective degrees of freedom at any T , we may model the chiral symmetry restoration transition with the help of Eq. (42).

Campbell *et al.* [50] have shown how to build the correct operator scaling properties into a low-energy effective Lagrangian in the Nambu-Goldstone (chirally-asymmetric) sector. Brown and Rho [51] have noted that this implies (in the chiral limit in which the bare (current) quark masses are zero) that

$$\frac{m_\rho^*}{m_\rho} = \frac{m_N^*}{m_N} = \frac{f_\pi^*}{f_\pi} = \dots = \Phi(T), \quad (45)$$

where $\Phi(T)$ describes the common scaling of all hadronic properties. Here the starred quantities denote in-medium values at finite temperature and/or density. The order parameter for chiral symmetry breaking is the effective pion decay constant, f_π^* [50, 51]. One can associate $f_\pi^* \rightarrow 0$ with the melting of the quark condensate, $\langle \bar{q}q \rangle^*$, with increasing temperature. The form of $\Phi(T)$ was justified in ref. [51]:

$$\Phi(T) = \left(\frac{\langle \bar{q}q \rangle^*}{\langle \bar{q}q \rangle} \right)^{1/3}. \quad (46)$$

We shall, however, use a different form of $\Phi(T)$, viz.,

$$\Phi(T) = \frac{\langle \bar{q}q \rangle^*}{\langle \bar{q}q \rangle}, \quad (47)$$

suggested by the Nambu model and shown by Koch and Brown [52] to model lattice gauge data better than (46). With the temperature dependence of the quark condensate found by Gerber and Leutwyler [18] and confirmed in our recent paper [43],

$$\frac{\langle \bar{q}q \rangle^*}{\langle \bar{q}q \rangle} = \left(1 - \frac{T^2}{T_\chi^2} \right), \quad T_\chi \simeq 190 \text{ MeV}, \quad (48)$$

Eq. (45) leads to the relation

$$m^* = m \left(1 - \frac{T^2}{T_\chi^2} \right) \quad (49)$$

for all hadronic species. With (49), Eq. (42) takes on the form

$$p = \frac{\pi^2}{30} T^4 + \frac{C' \pi^2}{180} T^4 \int_{0.5}^{1.2} dm m^3 \left(\frac{m^*}{T} \right)^2 K_2 \left(\frac{m^*}{T} \right), \quad (50)$$

which upon the integration reduces, through $C'(1.2^4 - 0.5^4)/4 = 44.5$, to

$$p = \frac{\pi^2}{90} T^4 \left[\frac{3}{2} \left(\frac{m_\pi}{T} \right)^2 K_2 \left(\frac{m_\pi}{T} \right) + 89 \frac{y^5 K_5(y) - x^5 K_5(x) - 6y^4 K_4(y) + 6x^4 K_4(x)}{x^4 - y^4} \right], \quad (51)$$

$$x \equiv \frac{1.2\Phi(T)}{T}, \quad y \equiv \frac{0.5\Phi(T)}{T},$$

where $\Phi(T)$ is given in (47), and we have recovered the pion contribution term with the finite pion mass. Temperature dependence of the effective degeneracy, $g_{eff} \equiv p/p_{SB}$, with $p_{SB} \equiv 47.5\pi^2/90 T^4$, as well as those of ρ/ρ_{SB} , with $\rho_{SB} \equiv 3p_{SB}$, and p/ρ ($\simeq c_s^2$), are shown in Figs. 1-3, respectively. One sees that in the model considered here, chiral symmetry restoration corresponds to a smooth crossover in the thermodynamic variables p and ρ . Let us compare these results with those of lattice gauge calculations. To date the most complete lattice gauge calculations on energy production for temperatures near T_χ are those obtained by Kogut *et al.* [53]. For calculations involving two low mass quarks, they find rapid increases in both the quark and gluon energy densities beginning at $\beta = 5.30$ on a 6×12^3 lattice (β is related to the inverse coupling constant, $\beta = 6/g^2$). By $\beta = 5.35$, the sum of the quark and gluon energy densities is essentially blackbody (and corresponds to that of an ultrarelativistic Stefan-Boltzmann gas), although in the case of gluons the accuracy of their calculation is not good enough. They find “a smooth $\rho_{u,d}/T^4$ curve with considerable suppression [compared to blackbody] in the transition region.” The

measured transition is a smooth crossover, in contrast to a sharper change obtained previously with smaller lattices. Earlier calculations of the behavior of the quark and gluon energy densities in this temperature region are summarized by Petersson [54] and, within their accuracy, are roughly consistent with the results of Kogut *et al.* [53].

This smooth crossover is in agreement with the results obtained in this paper for the model of the hadron to quark-gluon transition with the truncated resonance spectrum and the temperature dependent effective hadron masses. It is seen in Figs. 1-3 that the crossover begins at $T \sim 140$ MeV (corresponding to a dip in the T -dependence of p/ρ at those temperatures seen in Fig. 3 [1]), i.e., at $T \simeq 3/4 T_\chi$, and finishes at $T \sim T_\chi$, in agreement with the predictions made by Brown *et al.* [44]. At $T > T_\chi \simeq 190$ MeV, the system is essentially blackbody.

Although the results obtained in this paper are semiquantitative, they agree well with available lattice gauge data and reflect the main features of the latter,³ i.e., a smooth crossover in the thermodynamic variables taking place in a temperature range ~ 50 MeV, and the temperature dependent ratio ρ/ρ_{SB} approaching unity from above as the temperature increases.

4 Concluding remarks

We have obtained the equation of state of hot hadronic matter, by taking into account the contribution of the massive states with the help of the resonance spectrum $\tau(m) \sim m^3$ justified in our previous papers. This equation of state is in agreement with that provided by the low-temperature expansion for the pion interacting gas. We have shown that in this picture the deconfinement phase transition is absent, in agreement with lattice gauge calculations which show the only phase transition of chiral symmetry restoration. The latter was modelled with the help of the restriction of the number of the effective degrees of freedom in the hadron phase to that of the microscopic degrees of freedom in the quark-gluon phase, through the corresponding truncation of the hadronic resonance spectrum, and the decrease of the effective hadron masses with temperature, predicted by Brown and Rho. The results are in agreement with lattice gauge data, thereby confirming the prediction of Brown and Rho, and show a smooth crossover in the thermodynamic variables which begins at $T \simeq 3/4 T_\chi$ and finishes at $T \simeq T_\chi$, where $T_\chi \simeq 190$ MeV is the critical temperature for the chiral symmetry restoration transition, i.e., takes place in a temperature range ~ 50 MeV. At $T \cong T_\chi$ the system ends up in the chirally symmetric phase and is essentially blackbody.

³These features are seen, e.g., in Fig. 1 of the most recent review on lattice gauge calculations [55]: the crossover starts at $T \sim 150$ MeV and finishes at $T \sim 200$ MeV, the ratio ρ/ρ_{SB} approaches unity from above as the temperature increases. With the account for the error bars of this calculation, the system is essentially blackbody at T just above 200 MeV.

Although our results are in good agreement with lattice gauge data, we, along with the authors of ref. [44], have at present no way of how to decompose the hadron pressure (and energy density),

$$p = \frac{g_{eff}\pi^2}{90}T^4, \quad g_{eff} \rightarrow 47.5 \quad \text{with } T \rightarrow \infty,$$

into the separate quark and gluon components both above and below T_χ . Also, we are not clear about what happens above T_χ in a real world. This is the matter of subsequent investigations.

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FIGURE CAPTIONS

Fig. 1. The ratio p/p_{SB} , with p given in Eq. (51), as a function of temperature.

Fig. 2. The ratio ρ/ρ_{SB} as a function of temperature.

Fig. 3. The ratio p/ρ as a function of temperature.





